Answer for Q1

This problem is similar to *The Product Mix Problem* discussed in class, therefore we can solve the problem use the same approach:

Step 1. The decision variables

Let us denote the monthly production of Type A by x_1 tons, of Type B by x_2 tons, and of Type C by x_3 tons, as decision variables. Here, x_1, x_2 and x_3 are real numbers, which must be positive.

Step 2. The objective function

We would like to maximize our profit. Based on table, our profit per month = $210x_1 + 400x_2 + 600x_3$. Thus the objective function is to maximize the value of the function, $z = 210x_1 + 400x_2 + 600x_3$.

Step 3. The constraints

Since the supply of each raw material is limited, we need one constraint equation for each, as follows:

Pulp 1: $1.9x_2 + 2.4x_3 \leq 1500$

Pulp 2: $0.5x_2 \leq 800$

Pulp 3: $1.3x_1 + 2.1x_2 + 0.8x_3 \leq 500$

Pulp 4: $2.5x_1 \leq 2000$

Step 4. Complete the formulation

The problem is therefore completely specified as follows:

maximize $z = 210x_1 + 400x_2 + 600x_3$

subject to

$$1.9x_2 + 2.4x_3 \leq 1500$$
$$0.5x_2 \leq 800$$
$$1.3x_1 + 2.1x_2 + 0.8x_3 \leq 500$$
$$2.5x_1 \leq 2000$$
$$x_1, x_2 \text{ and } x_3 \geq 0$$

Answer for Q2

i) Let us rewrite the constraint for the available amount of potash as: $x + y \leq c$. Using the graphical solution from the notes, we confirm that the optimum solution is at the corner point determined by this constraint and the constraint on Urea, namely: $2x + y \leq 1500$. Since these two define the corner point, we can use the boundary of these two constraints to find the coordinate of the corner point, namely we can use the ***equations***: $x + y = c$, and $2x + y = 1500$. Solve:

$2x + y = 1500$ --- (1)

$x + y = c$ --- (2)

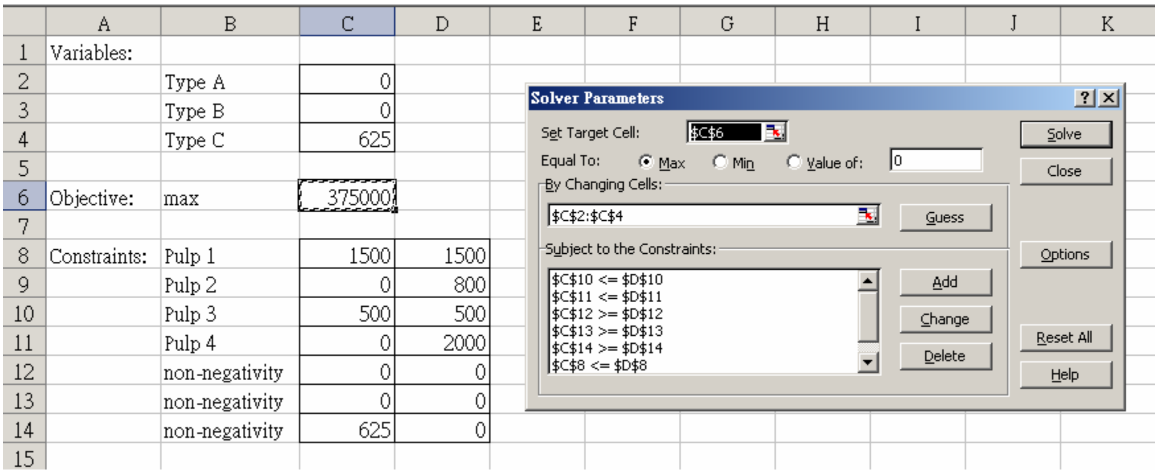
(1) - (2) gives: $x = 1500 – c$ --- (3)

Substituting x from (3) into (2) gives: $y = 2c – 1500$ --- (4)

Plug *x* and *y* from (3), (4) into the expression for the objective, $z = 15x + 10y$, to get: $z = 15(1500 – c) + 10(2c – 1500) = 7500 + 5c$ --- (5)
The derivative dz/dc gives the rate of change of profit for unit rate of change in the amount of potash; differentiating (5), we get:
$dz/dc = 5$. In other words, 1 unit change in potash will change the profit by $5.

ii) From the graphical solution, we can see that there is a finite slack in the amount of rock phosphate used (200 units of rock phosphate). Therefore a unit change in the availability of this resource will not affect the location of the corner point defining the optimum. Thus it has no impact in profit.

Answer for Q3



	A	B	C	D
1	Variables:			
2		Type A	0	
3		Type B	0	
4		Type C	625	
5				
6	Objective:	max	375000	
7				
8	Constraints:	Pulp 1	1500	1500
9		Pulp 2	0	800
10		Pulp 3	500	500
11		Pulp 4	0	2000
12		non-negativity	0	0
13		non-negativity	0	0
14		non-negativity	625	0
15				