APPENDIX: Derivation of the Newsvendor model optimal solution
[These notes are for reference only – not included in the exam]

In elementary calculus, we know that extreme points of a smooth, differentiable function f(x) can be obtained by solving the equation d f(x)/dx = 0. Here we consider a more complex case, when the function is an integral. A result in real analysis shows (the proof is omitted here), that if we have a function:

$F(y) = \int_{g(y)}^{h(y)} f(x,y)\,dx$, then its extreme values can be found by solving:

$$\frac{d}{dy}F(y) = \frac{d}{dy}\int_{g(y)}^{h(y)} f(x,y)\,dx \;=0= \int_{g(y)}^{h(y)} \frac{\partial f(x,y)}{\partial y}dx + f(h(y),y)\frac{d\,h(y)}{dy} - f(g(y),y)\frac{d\,g(y)}{dy}$$

In the case of the Newsvendor model, we wanted to find the minimum of:

$$g(Q) \;=\; E(G(Q)) \;=\; \int_{x=0}^{Q} c_0\,(Q-x)P(x)\,dx + \int_{x=Q}^{\infty} c_u\,(x-Q)P(x)\,dx$$

For the first term, y → Q, g(y) = 0 (a constant), h(y) → Q, and f(x, y) → *f(x, Q) =* c_0*(Q-x)P(x)*. Taking the derivative, we get:

$$\int_0^Q \frac{\partial c_0(Q-x)P(x)}{\partial Q}dx + f(Q,Q)\frac{d\,Q}{dQ} \;=\int_0^Q \frac{\partial c_0(Q-x)P(x)}{\partial Q}dx = \int_0^Q \frac{\partial c_0(Q-x)P(x)}{\partial Q}dx \,= \int_0^Q c_0\,P(x)\,dx$$

The analysis for the second term is similar, in this case, y → Q, g(y) = Q, h(y) → 0, and f(x, y) → *f(x, Q) =* c_u*(x-Q)P(x)*.

Thus the derivative, following the same steps, is: $\int_Q^{\infty} -c_u\,P(x)\,dx$

In other words, the expected cost is minimum when:

$$c_0\int_0^Q P(x)\,dx - c_u\int_Q^{\infty} P(x)\,dx \;=0.$$

Since P(x) is the probability density function, the integral in the first term is just the cumulative probability function, represents the probability that demand is Q or less. We denote this by F(Q). Similarly, the integral in the second term denotes the probability that the demand is larger than Q, which is (1 – F(Q)), since the total of these two probabilities must be 1. Therefore, at optimality,

c_0 F(Q) – c_u (1 – F(Q)) = 0
→ c_0 F(Q) = c_u (1 – F(Q))
→ (c_0 + c_u) F(Q) = c_u
→ F(Q) = c_u/(c_o + c_u) QED